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RESONANT CONFINEMENT OF PARTICLES WITH INTRINSIC MAGNETIC MOMENT IN A VARIABLE INHOMOGENEOUS MAGNETIC FIELD

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According to the Earnshaw theorem, a stable static configuration of electric charges is impossible. A similar statement is also true for particles having a magnetic moment in the absence of diamagnetic bodies [l]. However, electrodynamic confinement of charged particles is possible with no external feedback using combination of a constant and variable electric field [2].

Currently, traps of this type are rather widely used for suspending micron-sized particles (see, for example, [3]). A permanent magnet was suspended by combining a constant and variable magnetic field [4] and in a purely variable field [5]. The possibility of sustainable orbital motion of a spin particle in a variable and inhomogeneous constant magnetic field [6] is shown. In all cases (except for [6]), linearized equations of motion were considered, and the criterion of the suspension stability was the absence of a parametric resonance in the system.

However, the nonlinearity of equations of motion suggests the existence of the stable confinement in the presence of parametrically excited oscillations of a confined particle.

Consider, for example, a magnetic dipole being in a variable magnetic field produced by a solenoid, and a gravity field (Fig. 1). There are two instances: 1) the dipole is suspended above the end face of the solenoid; 2) the dipole is suspended inside the solenoid.

The distribution of the magnetic induction $\vec{B}$ (in a cylindrical coordinate system) is approximated as follows:

$$B_z = B_0(s + z^2 - 0.5\rho^2) \cdot \cos \omega\tau,$$

$$B_\rho = -B_0 \rho z \cdot \cos \omega\tau,$$

$$B_\varphi = 0,$$

$$s = const.$$

In the 1st instance, $\boldsymbol{S}>0$; in the second instance, $\boldsymbol{S}<0$.

The components of the vector potential of the field will then satisfy the Laplace equation. The dipole can be represented as a rotator with a mass *m*, moment of inertia $\boldsymbol{I}$, and magnetic moment $\vec{\rho}$. Neglecting the inhomogeneity of the field within a particle, the potential energy $\boldsymbol{U}$ can be written as follows:

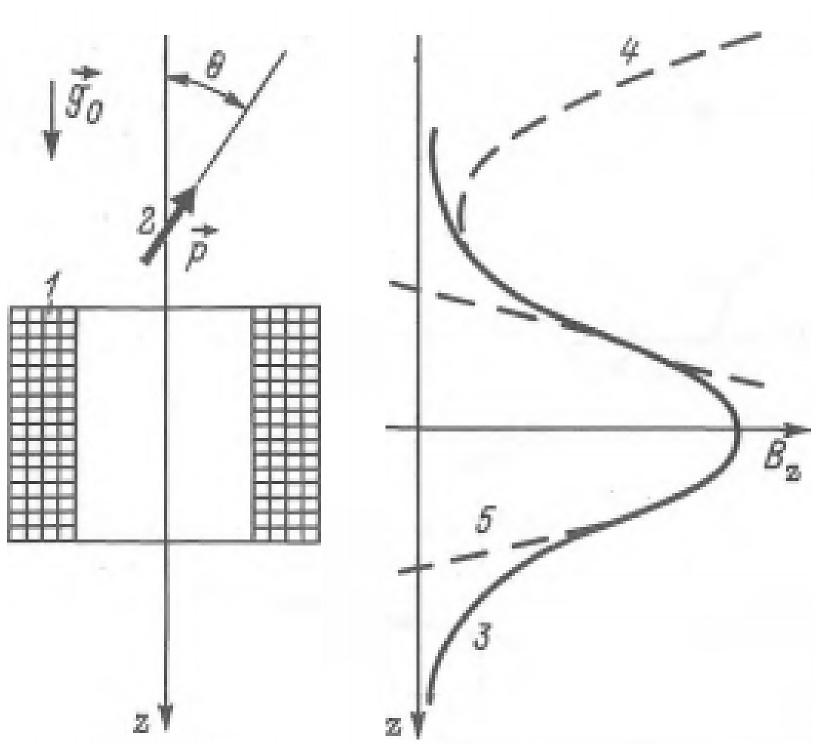


Fig. 1. 1 – solenoid, 2 – dipole, 3 – real relation $B_z = \varphi(z)$, 4 – approximation $\varphi(z)$ for the 1st instance, 5 – approximation $\varphi(z)$ for the 2nd instance.

$$U = -\vec{p} \cdot \vec{B} - m g_0 z,$$

wherein $\boldsymbol{g_0}$ is the acceleration of gravity.

The moment of momentum in respect to the coordinate φ is assumed to be equal to 0. Taking into account that $B_\varphi = 0$, the problem is reduced to a two-dimensional instance. In this event, the particle Lagrangian is written for the 1st instance as follows:

$$L=\dot{\rho}^2+\dot{z}^2+K\dot{\theta}^2+\varepsilon\left(\left(s+z^2-0.5\rho^2\right)\cdot\cos\theta-\rho z\cdot\sin\theta\right)\cdot\cos 2t+gz,$$

wherein $K=\frac{I}{m}$, $\varepsilon=\frac{8\rho B_0}{m\omega^2}$, $g=\frac{8g_0}{\omega^2}$, $t=\frac{\omega\tau}{2}$.

The Lagrangian for the second instance is written in a similar manner.

It is convenient to analyze the stability of solutions to the corresponding equations using the technique described under [7]. In this case, there is a search for approximate solutions in the form of a series $x=a_0+\sum_{k=1}^{n}\left(a_k\cdot\cos kt+\frac{b_k}{k}\cdot\sin kt\right)$.

As a result of the calculations, there are the equations $\dot{a}_\alpha=f(a_\beta, b_\beta, K, \varepsilon, g)$, $\dot{b}_\alpha=f(a_\beta, b_\beta, K, \varepsilon, g)$ (1), wherein $\alpha,\beta=0.1,\ldots,n$.

Critical points are obtained by solving the system of equations $\dot{a}_\alpha=0$, $\dot{b}_\alpha=0$. To determine the stability of the critical points, the equations (1) are examined for the stability in variations.

Such calculations were carried out for both instances when $|S|=1$.

Periodic solutions were sought in the form of:

$$z=a_{10}+a_{11}\cdot\cos t+b_{11}\cdot\sin t+a_{12}\cdot\cos 2t+b_{12}\cdot\sin 2t,$$

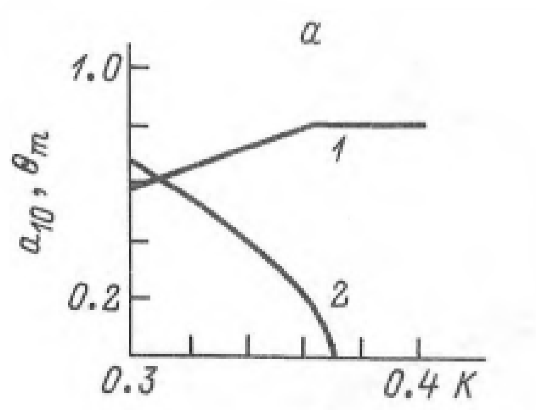
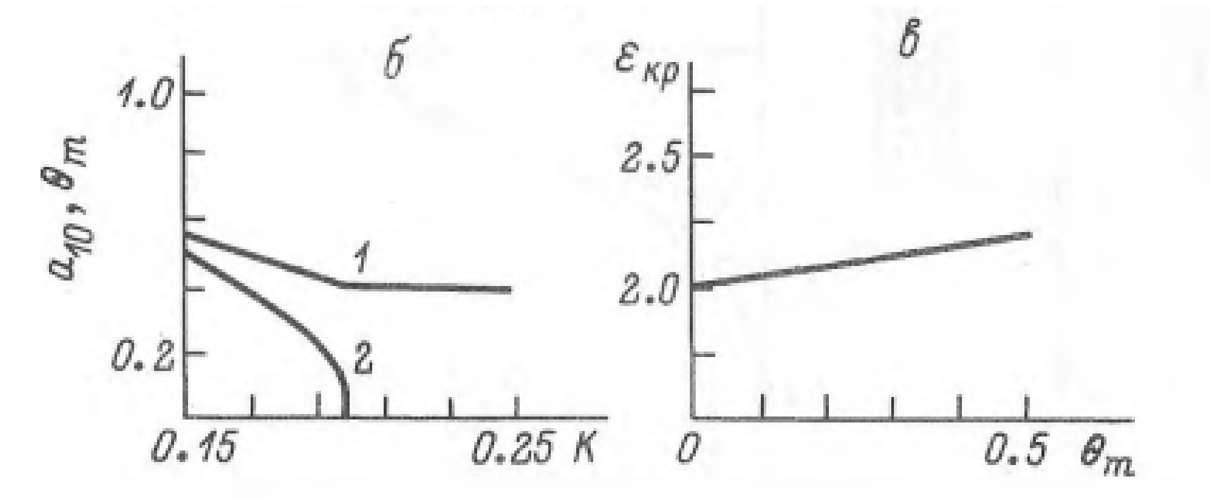


Fig. 2. $1 - a_{10}(K)$, $2 - \Theta_m(K)$.

a) 1st instance, $g = 0.2$, $\varepsilon=1$;
b) 2nd instance, $g = 0.1$, $\varepsilon=1$;
c) 2nd instance, $g = 0.1$.



$$\rho = a_{21} \cdot \cos t + b_{21} \cdot \sin t,$$

$$\theta = a_{31} \cdot \cos t + b_{31} \cdot \sin t.$$

In the calculations, it was assumed that $a_{ik}, b_{ik} \ll 1$, and the members above the 4-th order of smallness were discarded.

Relations of a_{10} and the amplitude of angular oscillations of the dipole $\theta_m = \sqrt{a_{31}^2 + b_{31}^2}$ to K obtained as a result of the calculations are shown in Fig. 2. As can be seen from the graphs, the spatial separation of particles according to the K value is possible. The solution becomes unstable when $\varepsilon > \varepsilon_{кр}$, in the 1st instance, $\varepsilon_{кр} = 2$ in the accepted approximation and it is independent of Θ_m; and in the 2nd instance, this dependence is present (Fig. 2). In particular, such a combination of K and g is possible, so that, when ε increases, the stable solution becomes unstable and then becomes stable again after oscillations of the dipole with respect to Θ take place.

In the 2nd instance, another kind of instability of the solution arises, it is associated with the fact that together with the increase of Θ_m the firmness of the suspension decreases up to negative values. In this case, the stationary point $z = 0$, which corresponds to the absence of gravity ($g = 0$), becomes unstable, and at $g \neq 0$ the coordinate of the suspension point increases without limit. For the 1st instance, a similar instability by the coordinate ρ can be observed.

To verify the numerical and analytical calculations, the equations of motion were simulated by means of the analog computing machinery "Rusalka" and the full-scale modeling was carried out. During this process, a magnet made of barium ferrite having 8 mm length was suspended over the end face of an electromagnet, which corresponds to the 1st instance. In order to expand the scope of initial conditions, which lead to the retention of the magnet, the suspension was carried out in glycerin. The amplitude of oscillations with respect to Θ was varied by imposition of an in-phase or anti-phase homogeneous variable magnetic field. In this case, the behavior of the magnet was described qualitatively by the solutions (Fig. 2), obtained as a result of calculations.

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