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Received by the Editors
Jul 22, 1985
In final version
April 2, 1986

UDC 53:51 **Technical Physics Journal, vol. 57, v. 4, 1981**

STABILITY OF STATIONARY MOTION OF A MAGNETIC WOLF IN A UNHOMOGENEOUS MAGNETIC FIELD

V. G. Shiroposov, V. M. Susloparov

When a macroscopic spin particle moves in an inhomogeneous magnetic field, conditions can arise when a stationary rotating magnetized sample will have stable trajectories in space.

The regular precession of a top with a fixed point in a constant uniform field has been well studied in the literature [1]. At the same time, the phenomenon of the emergence of stable states of motion in a system of two magnets oriented parallel to each other is known [2]. Using the classical approach [3,4] to the description of the spin motion, in [5] the effect of resonant capture of a particle with a magnetic moment in an alternating nonuniform magnetic field was obtained. Recent studies [6] on the stability of stationary orbital rotations of conducting bodies suspended in inhomogeneous magnetic fields testify to the possibility of such effects.

In this connection, the problem of the stability of the stationary orbital motion of a magnetic top in a constant axisymmetric inhomogeneous magnetic field is of particular interest. In the present work, in the form of such a top, a macroparticle is considered, which has a magnetic moment μ and its own rotation around its principal axis of inertia, which coincides with the vector μ (see picture). Due to the symmetry of the magnetic field $H \simeq (0, 0, H(r))$ relative to the *OZ* axis, we restrict ourselves to considering the motion of the top's center of mass in the *XOY* plane.

Neglecting the influence of dissipation on the motion of the top, we write down the preserved Hamilton function of the system

$$\mathscr{H} = \frac{m}{2}(\dot{r}^2 + r^2\dot{\varphi}^2) + \frac{I}{2}(\dot{\chi}^2 \sin^2\theta + \dot{\theta}^2) + \frac{I_0}{2}(\dot{\psi} + \dot{\chi}\cos\theta)^2 - \mu H(r)\cos\theta, \qquad (1)$$

where *m* - mass; I_0, *I* - moments of inertia of a symmetric top about its main and side axes (lines of angles $O'N \perp OZ', \; O'N \perp \mu$); $r, \; \varphi$ — polar coordinates of the center of mass *0'* of the top on the *XOY* plane; Euler angles θ, ψ and χ set the orientation of the axes of inertia of the top with respect to the rectangular coordinate system *O 'X' Y 'Z'*, associated with the center of inertia of the moving body.

Cyclic coordinates φ, χ and ψ correspond to the first three integrals of motion of the system

$$mr^2\dot{\varphi} = L_z,$$
$$I_0(\dot{\psi} + \dot{\chi}\cos\theta)\cos\theta + I\dot{\chi}\sin^2\theta = l_z,$$
$$I_0(\dot{\psi} + \dot{\chi}\cos\theta) = l_3, \tag{2}$$

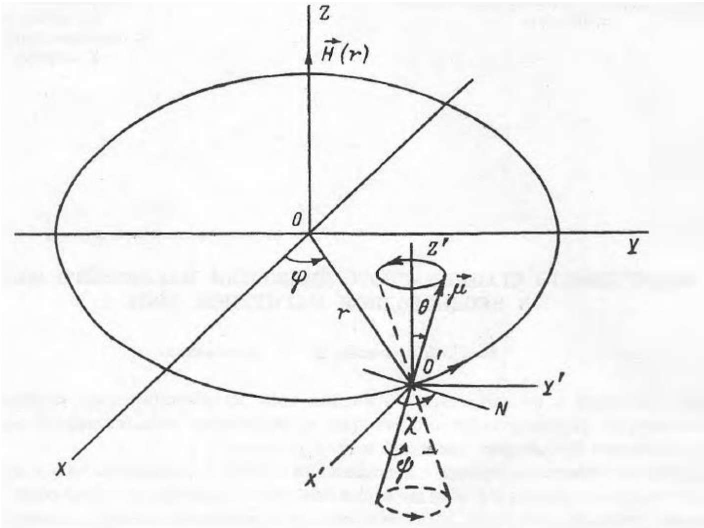
with the help of which one can similarly [1, p.143] exclude cyclic velocities $\dot{\varphi}$, $\dot{\chi}$ and $\dot{\psi}$, from the expression for energy (1). As a result, we get

$$E = \frac{m}{2}\dot{r}^2 + \frac{I}{2}\dot{\theta}^2 + U_{эфф}, \quad E = \mathscr{H} - \frac{l_3^2}{2I_0},$$

where is the effective potential energy of the system

$$U_{эфф} = -\mu H(r)\cos\theta - \frac{L_z^2}{2mr^2} + \frac{(l_z - l_3\cos\theta)^2}{2I\sin^2\theta} \tag{3}$$

is a function of two «positional» variables *r* and θ.



Along with complex motions described using nonlinear equations, the top can perform stationary motion - uniform motion along the orbit of radius $r_0\ (\dot{r} = 0)$ with simultaneous precession at a certain angle $\theta_0\ (\dot{\theta} = 0)$ to the axis OZ'. Finding the conditions of such an "orbital" procession is associated with finding extrema $U_{эфф}(r, \theta)$ at points r_0 and θ_0

$$\left(\frac{\partial U_{эфф}}{\partial r}\right)_0 = 0, \quad \left(\frac{\partial U_{эфф}}{\partial \theta}\right)_0 = 0, \tag{4}$$

which, using expression (3) for $U_{эфф}$, can be represented as

$$\mu\left(\frac{\partial H}{\partial r}\right)_0 \cos\theta_0 + \frac{L_z}{mr_0^3} = 0,$$
$$\mu H(r_0)\sin\theta_0 + \frac{(l_3 - l_z\cos\theta_0)(l_z - l_3\cos\theta_0)\sin\theta_0}{I\sin^4\theta_0} = 0. \tag{5}$$

The stability conditions for the considered stationary motion, according to Routh's theorem [7] and Sylvester's criterion, have the form

$$a_{rr} > 0, \quad a_{rr}a_{\theta\theta} - a_{r\theta}a_{\theta r} > 0, \tag{6}$$

Where

$$a_{rr} = \left(\frac{\partial^2 U_{эфф}}{\partial r^2}\right)_0 = -\mu\left(\frac{\partial^2 H}{\partial r^2}\right)_0 \cos\theta_0 + \frac{3L_z^2}{mr_0^4},$$

$$a_{r\theta} = \left(\frac{\partial^2 U_{эфф}}{\partial r \partial \theta}\right)_0 = \mu \left(\frac{\partial H}{\partial r}\right)_0 \sin\theta_0, \qquad a_{r\theta} = a_{\theta r},$$

$$a_{\theta\theta} = \left(\frac{\partial^2 U_{эфф}}{\partial \theta^2}\right)_0 = \frac{(l_z - l_3\cos\theta_0)^2 \sin^2\theta_0 + (l_3 + l_3\cos^2\theta_0 - 2l_z\cos\theta_0)^2}{I\sin^4\theta_0}.$$

The first equality in (6), taking into account (5), leads to the condition of radial stability of a particle in a central field [8]

$$\frac{(\partial^2 H/\partial r^2)_0}{(\partial H/\partial r)_0} r_0 > -3, \tag{7}$$

Which for an inhomogeneous magnetic field ... the dimensional coefficient) is reduced to the requirement ... Thus, within the framework of the model under consideration, the orbital motion of the top in magnetic fields of the type ... (magnetic dipole) is not stable.

The second inequality (6) determines, using relations (5), the range of values of the system parameters at which the given stationary motion of a magnetic particle is stable.

We can say that the implementation of such a motion corresponds to the synchronization of objects with close frequencies [9] For this, writing down the conditions for the feasibility of stationary motion (5) taking into account (2) in terms of the frequencies of orbital rotation $\omega_1=\dot{\varphi}$, regular precession $\omega_2=\dot{\psi}$ and proper rotation $\omega_3=\dot{\phi}$, we get

$$\mu\left(\frac{\partial H}{\partial r}\right)_0 \cos\theta_0 + m\omega_1^2 r_0 = 0,$$

$$[\omega_2^2(I_0 - I)\cos\theta_0 + I_0\omega_2\omega_3 + \mu H(r_0)]\sin\theta_0 = 0. \tag{8}$$

In the case of a magnetic rotator $(I_0=0)$ system (8) is reduced to the equation

$$(\omega_1^2\omega_2^2 - \omega_{L_0}^2\omega_{l_0}^2)\left[1 - \left(\frac{\omega_1}{\omega_{L_0}}\right)^4\right]^{1/2} = 0,$$

Where

$$\omega_{L_0}^2 = -\mu\left(\frac{\partial H}{\partial r}\right)_0 \frac{1}{r_0}, \quad \omega_{l_0}^2 = \mu H(r_0)/I, \quad \cos\theta_0 = (\omega_1/\omega_{L_0})^2.$$

At $\omega_1^2 \neq \omega_{L_0}^2$ (respectively $\theta_0 \neq 0, \pi$) we have $\omega_1^2\omega_2^2 = \omega_{L_0}^2\omega_{l_0}^2$, which is similar to the conditions for synchronization of objects with close frequencies [9].

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Received by the Editors
September 22, 1985
In final version
March 10, 1986

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