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THE TASK OF TWO MAGNETIC DIPOLES WITH ACCOUNTING EQUATIONS OF THE MOVEMENT OF THEIR SPINS

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The problem of two interacting magnetic dipoles with allowance for their spins is considered. The possibility of the appearance of stable discrete states of such a system (the absence of a collapse of dipoles) in the case of resonance is shown. Quantitative estimates of the stability of the system from macroscopic dipoles are given.

Usually a system of two magnetic dipoles is considered unstable [1-5]. The instability is due to a dipole-dipole interaction proportional to $\boldsymbol{r}^{-3}$. However, allowance for only the dipole-dipole interaction is not correct. Considering the problem of two magnetic dipoles, the authors, as a rule, consider them pointlike. This automatically leads to the rejection of the terms in the Lagrangian responsible for the appearance of the spin equation, the magnetic dipoles themselves, and to their collapse.

In the general case, the solution of the problem can be obtained from the system of equations-the spin equation

$$\mathrm{d}\boldsymbol{S}/\mathrm{d}t=(\mu/\hbar)[\boldsymbol{S}*\boldsymbol{H}], \tag{1}$$

and the equation of the force acting on a particle with an intrinsic magnetic moment $\boldsymbol{\mu}=\mu\boldsymbol{S}$ [6-8],

$$\boldsymbol{F}= - \mu\nabla(\boldsymbol{S}*\boldsymbol{H}), \tag{2}$$

either on the basis of a specific dipole model and the corresponding Lagrangian [2-5].

Considering various models, Kozorez [2-5] showed that taking magnetic moments above the dipole leads to the stability of the orbital motion of one magnetic particle around the other at distances comparable with their dimensions (Fig. 1). In the case of a single dipole located in a resonant inhomogeneous magnetic field with potential energy type $\boldsymbol{r}^{-3}$, taking into account the spin equation (1) leads to the appearance of stable states of motion [9-11]. For a system of two magnetic dipoles with spin, the appearance of an analogous resonance stability should also be expected without an external field, since the nature of their interaction at resonance essentially depends on the frequency detuning [9, 10]. Let us analyze the possibility of such stability.

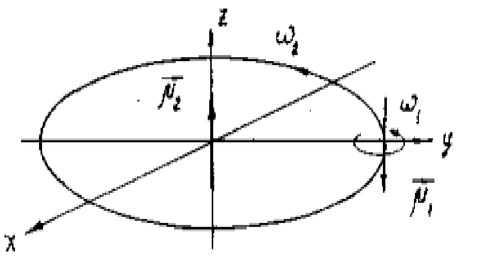


Fig.1. Stability in the system of two magnets according to Kozorez [2].

In the beginning, by analogy with the works of Kozorez ([2-5], Fig.1), we consider a system of two dipoles without dissipation. We confine ourselves to the case of motion of the magnetic dipole $\boldsymbol{\mu}_1$ in the field of a dipole $\boldsymbol{\mu}_2$ fixed by the translational degrees of freedom and placed at the origin. Then the equations of motion (1), (2) take the form:

$$\mathrm{d}\boldsymbol{\mu}_1/\mathrm{d}t= \gamma_1[\boldsymbol{\mu}_1*\boldsymbol{H}_{12}], \tag{3}$$

$$\mathrm{d}\boldsymbol{\mu}_2/\mathrm{d}t= \gamma_2[\boldsymbol{\mu}_2*\boldsymbol{H}_{21}], \tag{4}$$

$$d^2(m\boldsymbol{r}_{12})/dt^2=\nabla(\boldsymbol{\mu}\boldsymbol{H}_{12}), \qquad (5)$$

wherein $\boldsymbol{H}_{12}=(3\boldsymbol{n}_i(\boldsymbol{\mu}_j\boldsymbol{n}_i)-\boldsymbol{\mu}_j)/r^3$ - dipole field $\boldsymbol{\mu}_j$ at the location of the $\boldsymbol{\mu}_i$; $i,j=1,2$; $\boldsymbol{n}_i=\boldsymbol{r}_{ij}/r$, $r=|\boldsymbol{r}_{ij}|$.

The system of equations (3) - (5) is self-consistent, nonlinear, in partial derivatives. We shall seek its particular solution, assuming in advance the existence of a periodic solution with frequency **ω.**

Turning into a rotating coordinate system $\boldsymbol{\omega}\uparrow\uparrow\boldsymbol{oz}$, for (3), (4) we obtain:

$$d\boldsymbol{\mu}_{\text{в}i}/dt=\gamma_i[\boldsymbol{\mu}_{\text{в}i}*\boldsymbol{H}_{\text{в}ij}]+[\boldsymbol{\mu}_{\text{в}i}*\boldsymbol{\omega}]. \qquad (6)$$

Assuming a regime of steady-state oscillations and the condition $d\boldsymbol{\mu}_{\text{в}i}/dt=\boldsymbol{0}$ from (6) we have ($A_{ij}\mu_{\text{в}ij}=0$, где $\{\mu_{\text{в}ij}\}=\{\mu_{\text{в}1x}, \mu_{\text{в}1y}, \mu_{\text{в}2x}, \mu_{\text{в}2y}\}$, $A_{1j}=\{0,\alpha_1,0,1\}$, $A_{2j}=\{-\alpha_1,0,2,0\}$, $A_{3j}=\{0,1,0,\alpha_2\}$, $A_{4j}=\{2,0,0,-\alpha_2\}$):

$$\begin{pmatrix} 0 & \alpha_1 & 0 & 1 \\ -\alpha_1 & 0 & 2 & 0 \\ 0 & 1 & 0 & \alpha_2 \\ 2 & 0 & -\alpha_2 & 0 \end{pmatrix} \cdot \begin{pmatrix} \mu^{\text{в}}_{1x} \\ \mu^{\text{в}}_{1y} \\ \mu^{\text{в}}_{2x} \\ \mu^{\text{в}}_{2y} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad (7)$$

wherein $\alpha_i=[(\omega-\omega_{ij})/\omega_{ij}](\gamma_i/\gamma_j)=4;1$, $\omega_{ij}=\gamma_i\mu_{\text{в}jz}/r^3$ - frequency of the Larmor precession of the $\boldsymbol{i}$-th dipole in the field of the $\boldsymbol{j}$-th. The axis of the rotating coordinate system is chosen parallel to the vector r_{12}.

Equations (7) have solutions for

$$\alpha_1\alpha_2=(\omega-\omega_{12})(\omega-\omega_{21})=4;\ 1; \qquad (8)$$

$$\mu^{\text{в}}_{iy}=\alpha_i^{-1}\mu^{\text{в}}_{jy}=-\alpha_j\,\mu^{\text{в}}_{jy}; \qquad (9)$$

$$\mu^{\text{в}}_{ix}=2\alpha_i^{-1}\mu^{\text{в}}_{jx}=-(\alpha_j/2)\mu^{\text{в}}_{jx}; \qquad (10)$$

$$\mu^{\text{в}}_{iz}=\mu_{iz}. \qquad (11)$$

The results of solution (8) - (11) for particular cases of particles with identical spins and identical values of gyromagnetic numbers $(|\gamma_1|=|\gamma_2|,\ \mu_1=\mu_2)$ are presented in Table. 1. It follows from this that resonance capture, in principle, is possible without allowance for dissipation only on the harmonics of the Larmor precession frequency of the first dipole in the field of the second. The appearance of the solution $\omega=-\omega_{12}$ is due to the absence of dissipation (due to a spin flip and its precession in the opposite direction). This situation is typical when considering the motion of the magnetization vector under magnetic resonance conditions without dissipation [12].

In the system of three spin particles, two of which are the same, on the basis of Table. 1 (No. 1.5, 2.6, 4.8), we can conclude that there are no more than two resolved states with opposite spin orientations (↑-↑↓ - 1-8) at each level - ω_{12}, $2\omega_{12}$, $3\omega_{12}$.

The specific dependence of the effective interaction energy $\boldsymbol{U_{d\bullet d}}$ from $\boldsymbol{r}$ can be found from the law of conservation of the angular momentum the motion of a closed system: the field is a dipole,

$$\boldsymbol{S}_{п}+\boldsymbol{S}_1+\boldsymbol{S}_2+[\boldsymbol{r}_{12}*m\boldsymbol{\upsilon}]=\boldsymbol{L}=\mathbf{const}, \tag{12}$$

Where $\boldsymbol{S}_i = \boldsymbol{\mu}_i/\gamma_i$ - in the general case, the mechanical moment of dipoles (spin - for elementary particles, the moment of rotation - for magnetized gyroscopes). Approximately, we can assume, neglecting losses and radiation, for the case (8) - (11):

$$\mu_{1z}/\gamma_1+\mu_{2z}/\gamma_2 +mr^2\omega\cong L_z. \tag{13}$$

The results following from equation (13) are summarized in Table. 2 (for estimates, the value of $\boldsymbol{l_z}$ is of the order of $2\mu_0/3^{1/2}|\gamma|$ and the notation $r_0=\gamma^2m$, respectively $\omega_0=|\gamma|\mu_0/(r_0r^2)$, $|\omega|=|\gamma|\mu_0/r^3$. The emergence of a "singularity" at the point $r\sim r_0$ ($|\omega|\sim\omega_0$) was to be expected in the solution of the resonance problem, since the terms of the type of the $\boldsymbol{S}_{п}$ (12) responsible for the dissipation in the system were discarded, Usually the dissipative terms impose restrictions on the variation of the angle precession θ ($\mu_z =\mu_0\cos\theta$) to the value $\theta_m\cong(\pi - \omega_r/\omega_1)/2$.

Table 1

The calculated values of the parameters of two interacting magnetic dipoles with allowance for their spins

№	$\gamma_{12}\mu_{1,2z}$	ω/ω_{12}	$\mu^{в}_{1,2x}$	$\mu^{в}_{1.2y}$	α_1	α_2	$V_{д.д.}=<\mu_1H_{12}>\vert_{t=2\pi\omega}$
1	$\gamma_1=\gamma_2$, ↑↓	3	↑↑	0	2	2	$(2\mu_z^2-\mu_0^2)/r^3$
2		2	0	↑↓	1	1	$(3\mu_z^2-\mu_0^2)/2r^3$
3		0	–	–	–1	–1	
4		–1	↑↓	↑↓	–2	–2	μ_0^2/r^3
5	$\gamma_1=-\gamma_2$, ↑↑	3	↑↓	0	–2	–2	$-(2\mu_z^2-\mu_0^2)/r^3$
6		2	0	↑↑	–1	–1	$-(3\mu_z^2-\mu_0^2)/2r^3$
7		0	–	–	1	1	
8		–1	↑↑	0	2	2	$-\mu_0^2/r^3$
9	$\gamma_1=\gamma_2$, ↑↓	0	–	–	1	1	
10		i	–	–	Im	Im*	Imaginary solution
11		i	–		Im	Im*	Imaginary solution
12	$\gamma_1=-\gamma_2$, ↑↑	0	–	–	–1	–1	
13		i	–	–	Im	Im*	Imaginary solution
14		i	–	–	Im	Im*	Imaginary solution

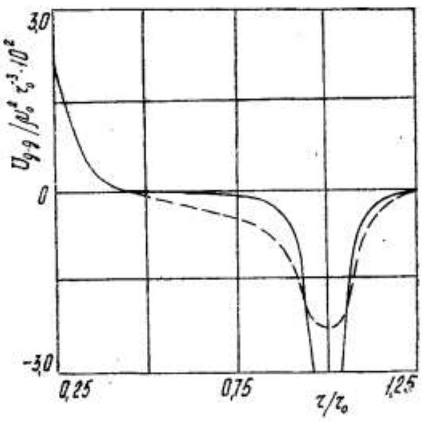
The corresponding graph of the dependence and a schematic diagram of possible stable motions are shown in Fig. 2, 3. As the dipoles approach, the precession angle θ increases, which should be expected due to an increase in the amplitude of the "pump" field $H_1=H_{x,y}^{д.д.}$, while in the point $\boldsymbol{r_0}$ decreases as a result of the synchronism of two frequencies-the "mechanical" $\omega_0^0(m)=|\gamma|\mu_0/r_0^3$ and the "magnetic" $|\omega|=|\gamma|\mu_0/r^3$.

Table 2

Values of the parameters of a system of interacting dipoles without allowance for dissipation

№	$\gamma_{12}\mu_{1,2z}$	ω/ω_{12}	$\sim\mu_z/\mu_o$	$V_{д,д}$
1	$\gamma_1=\gamma_2$, ↑↓	3	$\omega_0(\omega_0+\omega)$	$(2\mu_0^2/3r^3)[(1+3r_0/2r)^{-2}-3/2]$
2		2	$\omega_0(\omega_0+\omega)$	$(\mu_0^2/2r^3)[(1+r_0/2r)^{-2}-1]$
3		–1	$\omega_0(\omega_0-\omega)$	(μ_0^2/r^3)
4	$\gamma_1=-\gamma_2$, ↑↑	3	$\omega_0(\omega_0-\omega)$	$-(2\mu_0^2/3r^3)[(1-3r_0/2r)^{-2}-3/2]$
5		2	$\omega_0(\omega_0-\omega)$	$-(\mu_0^2/r^3)[(1-r_0/r)^{-2}-1]$
6		–1	$\omega_0(\omega_0+\omega)$	$-(\mu_0^2/r^3)$

Resonance capture in a system of one or two magnetic dipoles (pic. 1-3) refers to the tasks of synchronizing objects with close ones frequencies [13]. A complete rigorous analysis of the stability of solutions of such tasks should be carried out on the basis of involvement of methods of functions Lyapunov, Chetaev [14], by choosing specific models of dipoles [2-5] and the construction of appropriate langranganaks of closed matter-field system.



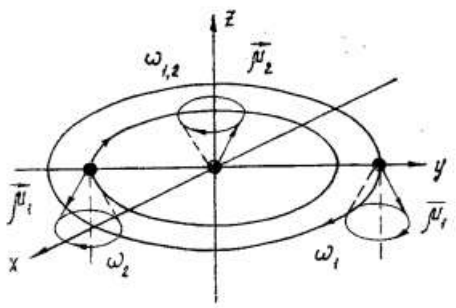


Fig. 2 Fig. 3

Fig. 2. Graphs of the averaged potential energy of two spin particles $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$, solid line - without dissipation, dotted line - taking into account dissipation (Table 1, No. 6)

Fig. 3. Possible stable motions of spin particles, $\boldsymbol{\omega_1} = 2\boldsymbol{\omega_{12}}$ (Table 1, No. 6), $\boldsymbol{\omega_2} = 3\boldsymbol{\omega_{12}}$ (Table 1, No. 5)

The remaining cases of resonance capture (1-3 of Table 2) can also lead to stability and a significant effect on the character of the motion of spin particles if we take into account additional terms of the type (2) of the Coulomb or gravitational type $\boldsymbol{1/r^2}$.

We estimate the parameters of the motion of dipoles for systems [4, 5], Table 2 (Figs 2, 3): the radius of the orbit r_0, the rotation frequency and precession of the dipole μ_1, the "lifetime" - the dissipation τ_r. As macrodipoles, let us take two spherical samples with the parameters: $4\pi M_0$=1750 Gs, ρ = 5 g/cm^3, d/2 = 0.1 cm and the frequency of self-rotation $\omega_c = 2\pi\cdot 10$ Hz, where $\mathbf{M_0}$ - magnetization, ρ - density, d - diameter of the samples. Accordingly, we obtain:

$$r_0/(d/2)\cong(4\pi M_0)^2/(3\pi\rho d^2\omega_c^{\,2})\cong 10^3, \tag{14}$$

$$\omega\cong 2\pi 10^{-5}\text{ Гц},\ |\gamma|=(r_0/m)^{1/2}\cong 2\pi 10\text{ Hz/э}, \tag{15}$$

$$\tau_r^{(4)}\cong(c/r_0\omega)^3(1/\omega)\cong 10^{45}\text{ c}, \tag{16}$$

$$\tau_r^{(5)}\cong(c/r_0\omega)^2\tau_r^{(4)}\cong 10^{71}\text{ c}, \tag{17}$$

wherein $\tau_r^{(4)}$ and $\tau_r^{(5)}$ is the dissipation time [15] in the system of radiating dipoles (Pic. 3). In the case of microdipoles, for example, the electron-positron, the radius of the orbit is exactly equal to the classical radius of the electron $r_0=\gamma^2_e m_e$= 2,8*10^{-13} cm, and further consideration on the basis of this approximate model loses its meaning.

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